10/10

Name: Ashley Hoh　　　　　　　　　　Date: Sept 7 2011

**Section 1.1 Quadratic Functions** $y=ax^2+bx+c$

1. Indicate the values of "a" "b" and "c" in each of the following equations:

| | | |
|---|---|---|
| a) $y=x^2-2x-5$ ✓ <br> $a=1$ $b=-2$ $c=-5$ | b) $y=\frac{1}{2}x^2+5$ ✓ <br> $a=\frac{1}{2}$ $b=0$ $c=5$ | c) $y-x^2+2=0$ <br> $y=x^2-2$ ✓ <br> $a=1$ $b=0$ $c=-2$ |
| d) $y=x(x-7)$ <br> $y=x^2-7x$ ✓ <br> $a=1$ $b=-7$ $c=0$ | e) $f(x)=x^2+1$ ✓ <br> $a=1$ $b=0$ $c=1$ | f) $y=-3(x+\frac{4}{3})^2-10$ <br> $y=-3(x^2+\frac{8}{3}x+\frac{16}{9})-10$ <br> $y=-3x^2-8x-\frac{16}{3}-10$ ✓ <br> $a=-3$ $b=-8$ $c=-\frac{46}{3}$ |

2. Factor each of the following quadratic functions and find i) the Coordinates of the Roots, ii) the Equation of the Axis of Symmetry, iii) Coordinates of the Vertex, iii) Domain and Range

| | | |
|---|---|---|
| a) $y=x^2+3x-18$ <br> $(x-3)(x+6)$ <br> Roots: $3,-6$ A of S: $x=-1.5$ <br> Vertex: $(-1.5,-11.25)$ ✗ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-11.25$ | b) $y=2x^2-x-2$ <br> Roots: $\frac{1+\sqrt{17}}{4}, \frac{1-\sqrt{17}}{4}$ A of S: $x=0.5$ <br> Vertex: $(0.5,-2)$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-2$ | c) $y=-x^2-12x-35$ <br> $(-x-5)(x+7)$ <br> Roots: $-5,-7$ A of S: $x=-6$ <br> Vertex: $(-6,1)$ <br> Domain: $x\in\mathbb{R}$ Range: $y\leq1$ |
| d) $y=x^2+\frac{5}{2}x-\frac{3}{2}$ <br> $(x+3)(x-\frac{1}{2})$ <br> Roots: $-3,\frac{1}{2}$ A of S: $x=-\frac{5}{4}$ <br> Vertex: $(\frac{5}{4},-\frac{49}{16})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{49}{16}$ | e) $y=6x^2+13x-5$ <br> $(3x-1)(2x+5)$ <br> Roots: $\frac{1}{3},-\frac{5}{2}$ A of S: $x=-\frac{13}{12}$ <br> Vertex: $(-\frac{13}{12},-\frac{289}{24})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{289}{24}$ | f) $y=15x^2-7x-2$ <br> $(5x+1)(3x-2)$ <br> Roots: $-\frac{1}{5},\frac{2}{3}$ A of S: $x=\frac{7}{30}$ <br> Vertex: $(\frac{7}{30},-\frac{169}{60})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{169}{60}$ |
| g) $y=32x^2-60x-27$ <br> $(8x+3)(4x-9)$ <br> Roots: $-\frac{3}{8},\frac{9}{4}$ A of S: $\frac{15}{16}=x$ <br> Vertex: $(\frac{15}{16},-\frac{441}{8})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{441}{8}$ | h) $y=\frac{1}{2}x^2+\frac{1}{2}x-6$ <br> $(\frac{1}{2}x+2)(x-3)$ <br> Roots: $-4,3$ A of S: $-\frac{1}{2}=x$ <br> Vertex: $(-\frac{1}{2},-\frac{49}{8})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{49}{8}$ | i) $y=x^2+\frac{1}{6}x-\frac{1}{6}$ <br> $(x+\frac{1}{2})(x-\frac{1}{3})$ <br> Roots: $-\frac{1}{2},\frac{1}{3}$ A of S: $-\frac{1}{12}$ <br> Vertex: $(-\frac{1}{12},-\frac{25}{144})$ <br> Domain: $x\in\mathbb{R}$ Range: $y\geq-\frac{25}{144}$ |

3. Graph each of the following quadratic functions and label the following: Roots, Axis of Symmetry, Vertex, and Y-intercepts:

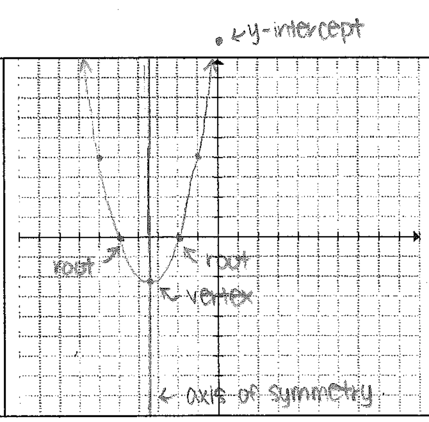
a) $f(x)=x^2+7x+10$

Roots: $-5,-2$

A of S: $x=-3.5$

Vertex: $(-3.5,-\frac{9}{4})$

Y-intercept: 10



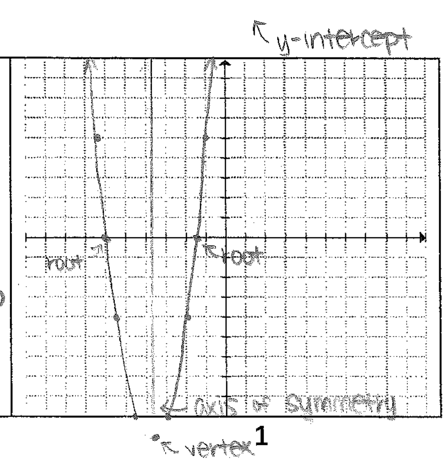

b) $f(x)=2x^2+15x+18$

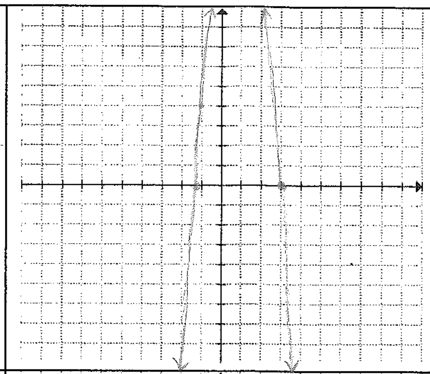
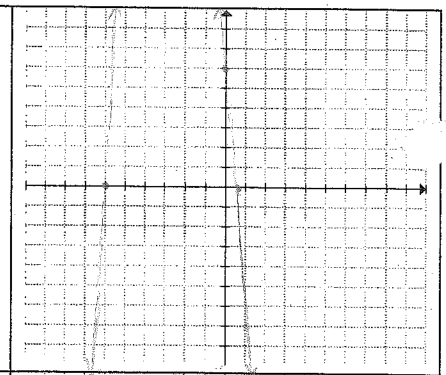
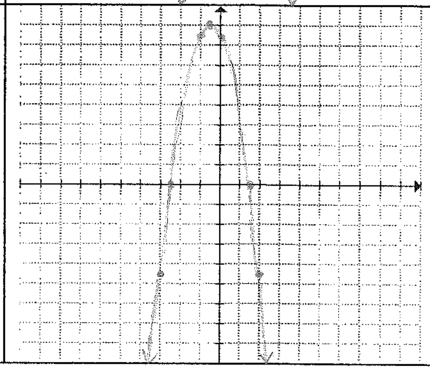
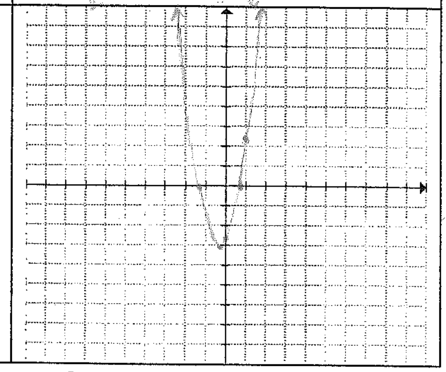
Roots: $-6,-\frac{3}{2}$

A of S: $x=-3.75$

Vertex: $(-3.75,-10.125)$

Y-intercept: 18



| | | | |
|---|---|---|---|
| c) $f(x)=12+5x-3x^2$ <br> $=-3x^2+5x+12$ <br> Roots: $-\frac{4}{3}$, $3$ <br> A of S: $x=\frac{5}{6}$ <br> Vertex: $(\frac{5}{6}, \frac{169}{12})$ <br> Y-intercept: $12$ |  | d) $f(x)=-2x^2-11x+6$ <br> Roots: $\frac{1}{2}$, $-6$ <br> A of S: $x=-\frac{11}{4}$ <br> Vertex: $(-\frac{11}{4}, \frac{169}{8})$ <br> Y-intercept: $6$ |  |
| e) $f(x)=\frac{15}{2}-2x-2x^2$ <br> Roots: $-\frac{5}{2}$, $\frac{3}{2}$ <br> A of S: $x=-\frac{1}{2}$ <br> Vertex: $(-\frac{1}{2}, 8)$ <br> Y-intercept: $\frac{15}{2}$ |  | f) $f(x)=-\frac{8}{3}+2x+3x^2$ <br> Roots: $\frac{2}{3}$, $-\frac{4}{3}$ <br> A of S: $x=-\frac{1}{3}$ <br> Vertex: $(-\frac{1}{3}, -3)$ <br> Y-intercept: $-\frac{8}{3}$ |  |

4. Solve each of the following quadratic equations. Provide your answers in exact form

| | |
|---|---|
| a) $5x-1=2x^2$ <br> $0=2x^2-5x+1$ <br> $\frac{5\pm\sqrt{25-8}}{4}=\frac{5\pm\sqrt{17}}{4}$ <br> $x=\frac{5+\sqrt{17}}{4}, \frac{5-\sqrt{17}}{4}$ | b) $8x+8=12x^2$ <br> $0=12x^2-8x-8$ <br> $\frac{8\pm\sqrt{64+384}}{24}=\frac{8\pm8\sqrt{7}}{24}$ <br> $=\frac{1\pm\sqrt{7}}{3}$ <br> $x=\frac{1+\sqrt{7}}{3}, \frac{1-\sqrt{7}}{3}$ |
| c) $x^2-5x+3=0$ <br> $\frac{5\pm\sqrt{25-12}}{2}=\frac{5\pm\sqrt{13}}{2}$ <br> $x=\frac{5+\sqrt{13}}{2}, \frac{5-\sqrt{13}}{2}$ | d) $6x+6=15x^2$ <br> $0=15x^2-6x+6$ <br> $0=5x^2-2x+2$ <br> $\frac{2\pm\sqrt{2-40}}{10}=\frac{2\pm\sqrt{38}}{10}$ <br> $=\frac{1\pm\sqrt{38}}{5}$ <br> $x=\frac{1+\sqrt{38}}{5}, \frac{1-\sqrt{38}}{5}$ |

5. Determine the value of the Discriminant and the Nature of the Roots:

| | | |
|---|---|---|
| a) $4x^2+10x+9=0$ <br> $100-144=-44$ <br> imaginary; $-44<0$ <br> or no real | b) $-x^2+6x+7=0$ <br> $36+28=64$ <br> 2 rational; $64=8^2$ | c) $-3x^2+\frac{1}{2}x+4=0$ <br> $\frac{1}{4}+48=\frac{193}{4}$ <br> 2 real; $\frac{193}{4}$ = positive |
| d) $5x^2-3x+\frac{1}{4}=0$ <br> $9-5=4$ <br> 2 rational; $4=2^2$ | e) $(x+3)^2=1$ <br> $x^2+6x+8=0$ <br> $36-32=4$ <br> 2 rational; $4=2^2$ | f) $\frac{x^2}{-3}=m$ <br> $m<0$ : 2 distinct roots <br> $m=0$ : 2 equal roots <br> $m>0$ : no real roots |
| g) $200+33x+x^2=0$ <br> $1089-800=289$ <br> 2 rational; $289=17^2$ | h) $0=x^2+12x-85$ <br> $144+340=484$ <br> 2 rational; $484=22^2$ | i) $0=3x^2-12x-288$ <br> $144+3456=3600$ <br> 2 rational; $3600=60^2$ |

6. Determine the vertex of the parabola $y = 3(x-20)(x+22)$

$0 = 3(x-20)(x+22)$

$(x-20)=0 \quad (x+22)=0$

$x=20 \quad x=-22$

$\boxed{\text{roots: } 20, -22}$

$\frac{20-22}{2} = -\frac{2}{2} = -1$

$\boxed{\text{A of S: } -1}$

$y = 3(-1-20)(-1+22)$

$y = 3(-21)(21)$

$\boxed{y = -1323}$

vertex: $(-1, -1323)$

7. A pebble is dropped from a bridge into a river at height "h" meters above. Let "t" be the number of seconds after the release. If $h(t) = 65 - 4.9t^2$, then how high is the pebble after 3 seconds? What is the domain and range of this scenario? When will the pebble hit the ground?

domain: $t \geq 0$

range: $0 \leq y \leq 65$

$h(3) = 65 - \frac{49}{10}(9)$

$h(3) = \frac{650-441}{10}$

$h(3) = \frac{209}{10}$

$h(3) = 20.9$

$0 = -\frac{49}{10}t^2 + 65$

$\frac{49}{10}t^2 = 65$

$t^2 = \frac{650}{49}$

$t = \frac{5\sqrt{26}}{7} \approx 3.64$

the pebble is 20.9m after 3 sec

the pebble will hit the ground at around 3.64 sec.

8. A tennis ball is dropped from a balcony. The height of the ball (h) above the ground is given by the formula $h(t) = 78.4 - 4.9t^2$. Where "t" is the number of seconds after release. How high is the balcony from the ground? When will the ball hit the ground?

the balcony is 78.4 units from the ground.

$\frac{49}{10}t^2 = 78.4 = \frac{392}{5}$

$t^2 = 16$

$t = 4$

the ball will hit the ground after 4 seconds.

9. Tom throws a football from the top of his building. The height of the ball is given by the formula: $h(t) = -3t^2 + 60t + 132$, where "h" is the height of the football and "t" is the number of seconds after the throw. What is the domain and range of this scenario? When will the ball be falling to 50m?

domain: $t \geq 0$

range: $0 \leq y \leq 132$

$50 = -3t^2 + 60t + 132$

$3t^2 - 10t - 132 = 0$

$t = \frac{5 + \sqrt{421}}{3} \approx 8.51$

the ball will fall to 50m after 8.51 seconds.

10. If the quadratic equation $(x-2)^2 + k = 0$ has <u>two distinct real roots</u>, then what is the range of "k"? (Multiple choice, cirlce one) Justiy your answer.

if the discriminant is positive

a) $k > 2$ b) $k < 0$ c) $k \leq 0$ d) $k \leq 4$

$(x-2)^2 + k = 0$

$\underbrace{x^2}_{a} \underbrace{-4x}_{b} \underbrace{+4+k}_{c} = 0$

$b^2 - 4ac$

$\rightarrow 16 - 4(4+k)$

$= 16 - 16 - 4k$

$-4k = $ positive

$\therefore k$ must be negative

11. Find the values of "A" and "B" if $x^2 + 6x + 16 = (x+A)^2 + B$

$x^2 + 6x + 16 = x^2 + 2xA + A^2 + B$

$x(x+6) + 16 = x(x+2A) + A^2 + B$

$x + 6 = x + 2A$

$\boxed{A = 3}$

$A^2 + B = 16$

$9 + B = 16$

$\boxed{B = 7}$

12. Find the values of "A" and "B" if $x^2 - 10x + 27 = (x + A)^2 + B$

$x^2 - 10x + 27 = x^2 + 2xA + A^2 + B$

$x(x-10) + 27 = x(x+2A) + A^2 + B$

$x - 10 = x + 2A$

$2A = -10$

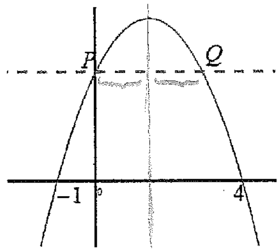
$\boxed{A = -5}$

$27 = A^2 + B$

$27 = 25 + B$

$\boxed{B = 2}$

13. The figure below shows the graph of $y = -x^2 + px + q$. The graph cuts the y-axis at point "P". A horizontal line is drawn through points "P" and "Q". What are the coordinates of point "Q"?




roots $= -1, 4$ (given)

A of S $= (-1+4)\frac{1}{2} = \frac{3}{2} = x$

P(0, q)

↳ y-intercept

$2x = \frac{6}{2} = 3$

Q (3, q)

14. If the quadratic function $y = ax^2 + bx + c$ has two equal roots and opens up, then which of the following statements are correct?

i) $a > 0$ ii) $c > 0$ iii) $b^2 - 4ac > 0$

*root opens up

$(x-z)^2 = x^2 - 2zx + z^2$

$(x+z)^2 = x^2 + 2zx + z^2$

* two equal roots

15. If $y = (x-2)^2$ and $y = 2x + 1$ intersect at points $(x_1, y_1)$ and $(x_2, y_2)$, then which of the following quadratic functions has the roots at $x_1$ *and* $x_2$?

a) $y = x^2 - 6x + 3$ b) $y = x^2 - 2x + 3$ c) $y = x^2 - 6x + 1$ d) $y = x^2 - 2x + 1$

$y = x^2 - 4x + 4$

$y = 2x + 1$

$x^2 - 4x + 4 = 2x + 1$

$0 = x^2 - 6x + 3$

16. Determine all values of "k" with $k \neq 0$ for which the parabola has its vertex on the x-axis.

$y = kx^2 + (5k+3)x + (6k+5)$

$y = kx^2 + 5kx + 3x + 6k + 5$

$a = k$ $b = 5k+3$ $c = 6k+5$

$b^2 - 4ac = 0$

$(5k+3)(5k+3) - 4$

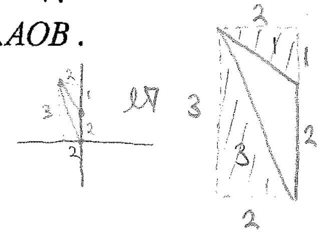
17. Point "A" is the vertex of the parabola $y = x^2 + 2$, point "B" is the vertex of the parabola $y = x^2 - 6x + 7$, and "O" is the origin. Determine the area of $\triangle AOB$.

$y = x^2 + 2$

vertex (0, 2)

$y = x^2 - 6x + 7$

vertex (3, −2)




$6 - 4 = 2$

area of $\triangle AOB$ is 2 units$^2$

18. Consider the function $f(x) = 2x^2 - 4x + c$. What value of "c" maximizes the product of the roots of the function, given that at least one root is real?

$2x^2 - 4x + c = 0$

$a = 2 \quad b = -4 \quad c = c$

when at least one root is real -

$*\, b^2 - 4ac > 0$

or

$*\, b^2 - 4ac = 0$

$16 - 8c > 0$

$16 > 8c$

$\boxed{c < 2}$

$16 - 8c = 0$

$8c = 16$

$\boxed{c = 2}$

based on trial and error..

$c = 2$

19. The parabola $y = f(x) = x^2 + bx + c$ has vertex "P" and the parabola $y = g(x) = -x^2 + dx + e$ has vertex "Q", where "P" and "Q" are distinct points. The two parabolas also intersect at "P" and "Q".

i) Prove that $2(e - c) = bd$.

$f(x) = x^2 + bx + c$

roots: $x = \dfrac{-b \pm \sqrt{b^2 - 4c}}{2}$

A of S: $-\frac{b}{2} = x$

vertex: $(-\frac{b}{2})^2 + b(-\frac{b}{2}) + c$

$= \frac{b^2}{4} - \frac{b^2}{2} + c$

$= -\frac{b^2}{4} + c$

$\boxed{P(-\frac{b}{2}, -\frac{b^2}{4} + c)}$

$g(x) = -x^2 + dx + e$

$-\frac{b^2}{4} + c = -(-\frac{b}{2})^2 + d(-\frac{b}{2}) + e$

$-\frac{b^2}{4} + c = -\frac{b^2}{4} - \frac{db}{2} + e$

$\frac{bd}{2} = e - c$

$bd = 2(e - c)$

ii) Prove that the line through points "P" and "Q" has slope $\frac{1}{2}(b + d)$ and y-intercept $\frac{1}{2}(c + e)$

OTHER CONT. ON SHEET

$g(x) = -x^2 + dx + e$

roots: $x = \dfrac{-d \pm \sqrt{d^2 + 4e}}{-2}$

A of S: $\frac{d}{2} = x$

vertex: $-(\frac{d}{2})^2 + d(\frac{d}{2}) + e$

$= -\frac{d^2}{4} + \frac{d^2}{2} + e$

$= \frac{d^2}{4} + e$

$\boxed{Q(\frac{d}{2}, \frac{d^2}{4} + e)}$

$P(-\frac{b}{2}, -\frac{b^2}{4} + c) \Rightarrow$ solved above

$m = \dfrac{(\frac{d^2}{4} + e) - (-\frac{b^2}{4} + c)}{(\frac{d}{2}) - (-\frac{b}{2})} = \dfrac{\frac{d^2}{4} + e + \frac{b^2}{4} - c}{\frac{d+b}{2}} = \dfrac{\frac{d^2}{2} + \frac{b^2}{2} + 2(e-c)}{d+b} = \dfrac{\frac{d^2+b^2}{2} + bd}{d+b} = \dfrac{d^2 + 2bd + b^2}{(d+b)2} = \dfrac{(d+b)^2}{2(d+b)}$

20. The equation $y = x^2 + ax + a$ represents a parabola for all real values of "a". Prove that each of thees parabolas pass through a common point and determine the coordinates of this point.

$y = x^2 + ax + a$

$x = -1$

$y = (-1)^2 - a + a$

$y = 1$

$\therefore (-1, 1)$

$y = x^2 + bx + b$

$y = x^2 + cx + c$

$x^2 + bx + b = x^2 + cx + c$

$b(x+1) = c(x+1)$

$b(x+1) - c(x+1) = 0$

$(b - c)(x + 1) = 0$

$x + 1 = 0$

$x = -1$

$y = 1 - b + b$

$y = 1$

$\therefore (-1, 1)$

ii) The vertices of the parabolas in part (a) lie on a curve. Prove that this curve is itself a parabola whose vertex is the common point found in part (a)

$y = x^2 + ax + a$

$x = \dfrac{-a \pm \sqrt{a^2 - 4a}}{2}$

$x = -\frac{a}{2}$ (of vertex)

$y = (-\frac{a}{2})^2 + a(-\frac{a}{2}) + a$

$= -\frac{a^2}{4} + a$

$\hookrightarrow -\frac{a^2}{4} \leftarrow$ quadratic

Name: Ashley Hoh 10 Date: Sept 13 2011

## Section 1.2 Quadratic Functions in Standard Form $y=a(x-p)^2+q$

1. Indicate the values of "a" "p" and "q" in each of the following equations:

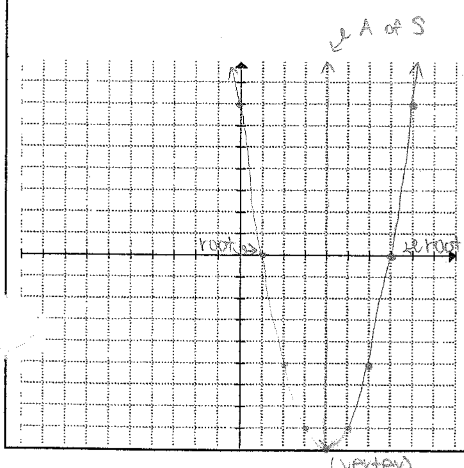
| | | |
|---|---|---|
| a) $y=3(x-4)^2+8$<br>$a=3$ $p=4$ $q=8$ | b) $y=2(x+6)^2-13$<br>$a=2$ $p=-6$ $q=-13$ | c) $y=-4x^2+10$ $=-4(x-0)^2+10$<br>$a=-4$ $p=0$ $q=10$ |
| d) $y=(-3x)^2+2$ $=9x^2+2$ $=9(x-0)^2+2$<br>$a=9$ $p=0$ $q=2$ | e) $y=(5x-20)^2$ $=25x^2-200x+400$ $=25(x^2-8x+16)=25(x-4)^2$<br>$a=25$ $p=4$ $q=0$ | f) $y=\dfrac{4(2x-2)^2-8}{8}+1$ $=2(x-1)^2$<br>$a=2$ $p=1$ $q=0$ |

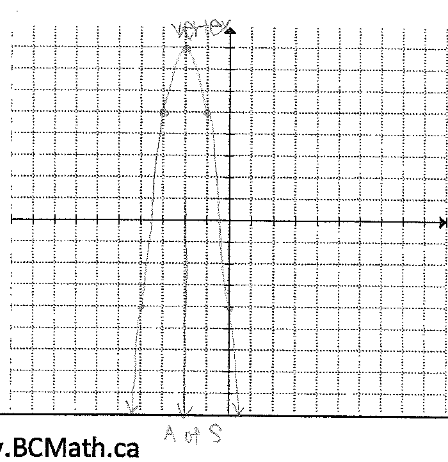
2. Factor each of the following quadratic functions and find i) the Coordinates of the Roots, ii) the Equation of the Axis of Symmetry, iii) Coordinates of the Vertex, iii) Domain and Range

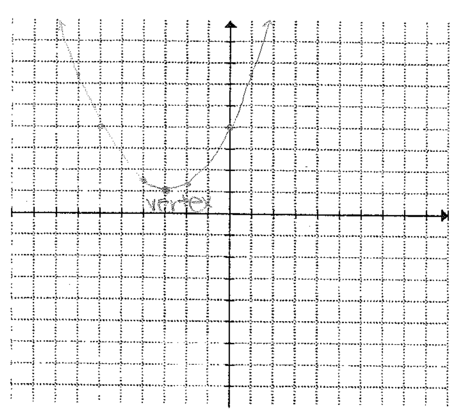
| | | |
|---|---|---|
| a) $y=x^2-5$<br>*Roots*: $\sqrt{5},-\sqrt{5}$ *A of S*: $x=0$<br>*Vertex*: $(0,-5)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq-5$ | b) $y=-2(x+2)^2$<br>*Roots*: $-2$ *A of S*: $x=-2$<br>*Vertex*: $(-2,0)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\leq0$ | c) $y=5(x-5)^2-10$<br>*Roots*: $5+\sqrt{2},5-\sqrt{2}$ *A of S*: $x=5$<br>*Vertex*: $(5,-10)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq-10$ |
| d) $y=7x^2-14$<br>*Roots*: $\sqrt{2},-\sqrt{2}$ *A of S*: $x=0$<br>*Vertex*: $(0,-14)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq-14$ | e) $y=(4x-4)^2-10$ $=16(x-1)^2-10$<br>*Roots*: $\frac{4+\sqrt{10}}{4},\frac{4-\sqrt{10}}{4}$ *A of S*: $x=1$<br>*Vertex*: $(1,-10)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq10$ | f) $y=5(3x)^2$ $=5(9x^2)=45x^2=45(x)^2$<br>*Roots*: $0$ *A of S*: $x=0$<br>*Vertex*: $(0,0)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq0$ |
| g) $y=\dfrac{(5x-5)^2+15}{5}$ $=5(x-1)^2+3$<br>*Roots*: / *A of S*: $x=1$<br>*Vertex*: $(1,3)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\geq3$ | h) $y=-2(3-x)^2-14$ $=-2(x-3)^2-14$<br>*Roots*: / *A of S*: $x=3$<br>*Vertex*: $(3,-14)$ *Domain*: $x\in\mathbb{R}$<br>*Range*: $y\leq-14$ | i) $y=\dfrac{2\sqrt{(x^4+4x^2+16)}+4}{-2}-1$<br>*Roots*: *A of S*:<br>*Vertex*: *Domain*: $x\in\mathbb{R}$<br>*Range*: |

3. Graph each of the following quadratic functions and label the following: Roots, Axis of Symmetry, Vertex, and Y-intercepts:

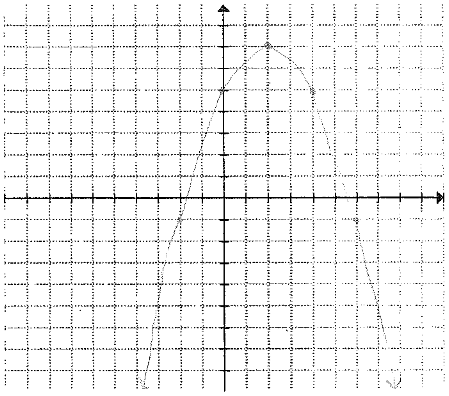
| | | |
|---|---|---|
| a) $y=(x-4)^2-9$ | b) $y=-3(x+2)^2+8$ | b) $y=\frac{1}{3}(x+3)^2+1$ |



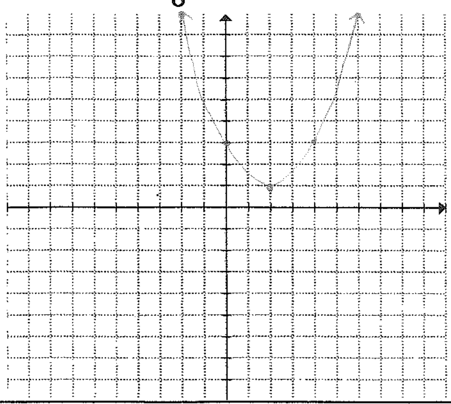



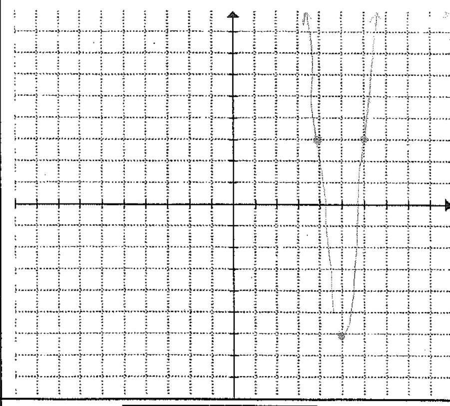
c) $y=-\frac{1}{2}(x-2)^2+7$



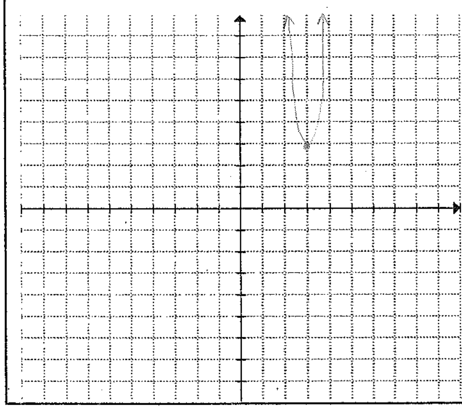
d) $y=\frac{(2x-4)^2+8}{8}$ $=\frac{1}{2}(x-2)^2+1$



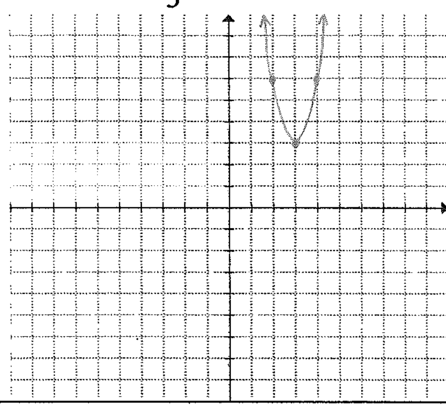
e) $y=(3x-15)^2-6$ $=9(x-5)^2-6$



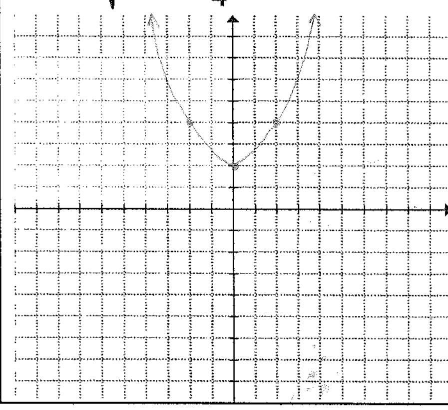
e) $y=2(2x-6)^2+3$ $=8(x-3)^2+3$



f) $y=\frac{(3x-9)^2+3}{3}+2$ $=3(x-3)^2+3$

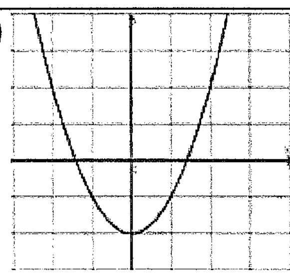


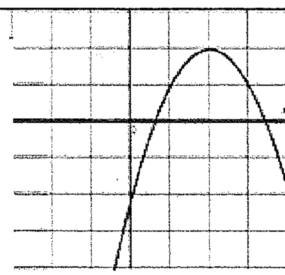
g) $y=\sqrt{\frac{(x^4-8x^2+16)}{4}}+4$ $=\frac{1}{2}(x)^2+2$

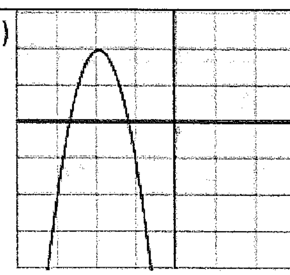


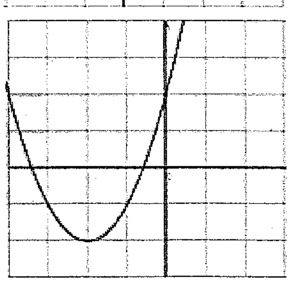
4. If each parabola is in the form of $y=a(x-p)^2+q$, then which graph best describes each equation:

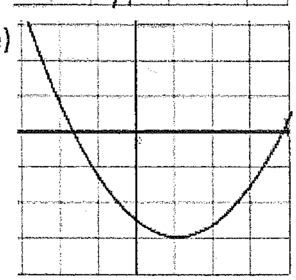
| | |
|---|---|
| i) $a<-1, p<0, q>0$ | f |
| ii) $0<a<1, p>0, q<0$ | e |
| iii) $a>0, p=0, q<0$ | a |
| iv) $0>a>-1, p<0, q>0$ | c |

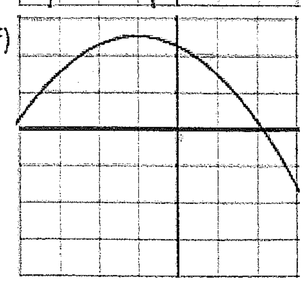
a) 

b) 

c) 

d) 

e) 

f) 

5. Convert the function $y=\frac{1}{2}x^2-4x+1$ into standard form.

$y=\frac{1}{2}(x^2-8x+2)$

$y=\frac{1}{2}[(x-4)^2-14]$

$y=\frac{1}{2}(x-4)^2-7$

6. If a ball is thrown upward from a height of 4 metres with an initial veloctiy of 6 m/s, its height, H(t), after t seconds is given by the equation $H(t) = -0.5t^2 + 6t + 4$. Determine the maximum height of the ball.

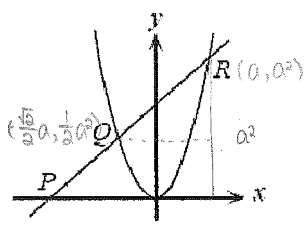
$H(t) = -\frac{1}{2}t^2 + 6t + 4$     vertex $(6, 22)$

$= -\frac{1}{2}(t^2 - 12t) + 4$

$= -\frac{1}{2}(t-6)^2 + 18 + 4$     the max height is 22m //

$= -\frac{1}{2}(t-6)^2 + 22$

7. A line with slope 1 passes through the point "P" on the negative x-axis as shown and intersects the parabola $y = x^2$ at points Q and R. If $PQ = RQ$, then what is the y-intercept of line PR?




y-coord of Q $= \frac{1}{2}a^2$

$\frac{1}{2}a^2 = x^2$

$x = \sqrt{\frac{1}{2}}\,a$

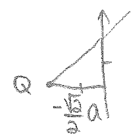
$a^2 - \frac{1}{2}a^2 = a + \sqrt{\frac{1}{2}}\,a$

$\frac{1}{2}a^2 = a + \sqrt{\frac{1}{2}}\,a$

$a^2 = 2a + 2\sqrt{\frac{1}{2}}\,a$

$a = 2 + 2\sqrt{\frac{1}{2}}$

$\boxed{a = 2 + \sqrt{2}}$





$\frac{\sqrt{2}}{2}a \rightarrow \frac{\sqrt{2}}{2}(2+\sqrt{2})$

$\boxed{= \sqrt{2} + 1}$

$\frac{1}{2}a^2 = \frac{1}{2}(2+\sqrt{2})^2 = \boxed{3 + 2\sqrt{2}}$

$b = (\sqrt{2} + 1) + (3 + 2\sqrt{2})$

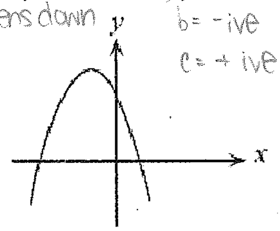
$= 4 + 3\sqrt{2}$ //

8. The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following statements below must be positive?

a) $a$    b) $bc$    c) $ab^2$    d) $b - c$    e) $c - a$

opens down    $b = -$ive    $a = -$ive

$c = +$ ive




$y = ax^2 + bx + c$

$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

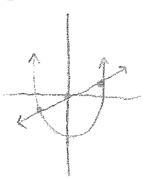
$\frac{b}{2a} = +$ ive

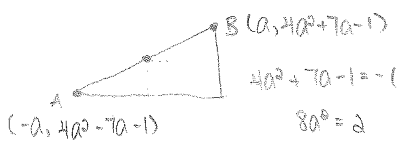
since $a = -$ive ∴ $b = -$ive

9. Consider the parabola . The value of the real number "c" for which such a parabola touches the x-axis exactly once is:

a) $-\frac{4}{5}$    b) 0    c) $\frac{2}{5}$    d) $\frac{4}{5}$    e) $\frac{\sqrt{5}}{4}$

10. Point "A" and "B" are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of $\overline{AB}$. What is the length of $\overline{AB}$?






$4a^2 + 7a - 1 = -(4a^2 - 7a - 1)$

$8a^2 = 2$

$a^2 = \frac{1}{4}$    $a = \pm\frac{1}{2}$

$\frac{1}{2}\overline{AB} = \sqrt{a^2 + (4a^2 + 7a - 1)^2}$

$\frac{1}{2}\overline{AB} = \sqrt{\frac{1}{4} + (1 + \frac{7}{4} - 1)^2}$

$\frac{1}{2}\overline{AB} = \sqrt{\frac{1}{4} + \left(\frac{49}{4}\right)} = \sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2}$

$\overline{AB} = 5\sqrt{2}$ //

11. The parabola $y = x^2 - 2x + 4$ is moved *'p'* units to the right and *"q"* units down. The x-intercepts of the resulting parabola are 3 and 5. What are the values of *"p"* and *"q"*?

$0 = (x - 1 - b)^2 + 3 - c$

$y = x^2 - 2x + 4$    let "p" be b

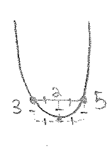
$y = (x-1)^2 + 3$    let "q" be c    $5 - 3 = 2$

$a = 1$   $p = 1$   $q = 3$    $y = (x - (1+b))^2 + (3 - c)$    since $a = 1$..

3 right    "p" = 3

4 down    "q" = -4 //

$a = 1$   $p = 1 + b$   $q = 3 - c$



12. If $y=a(x-2)^2+c$ and $y=(2x-5)(x-b)$ represents the same quadratic function, what is the value of the constant "b"

$a(x-2)^2+c=(2x-5)(x-b)$

$a(x^2-4x+4)+c=2x^2-5x-2bx+5b$

$ax^2-4ax+4+c=2x^2-(5+2b)x+5b$

$ax^2=2x^2$

$a=2$

$-4ax=-(5+2b)x$

$-8=-5-2b$

$-3=-2b$

$b=\frac{3}{2}$

13. the parabola $y=ax^2+bx+c$ has vertex $(p,p)$ and y-intercept $(0,-p)$, where $p\neq 0$, what is the value of "b"? a) $-p$ b) 0 c) 2 D) 4 E) $p$

$y=ax^2+bx+c$

$y=a(x^2+\frac{b}{a}x)+c$

$y=a(x^2+\frac{b}{a}x+\frac{b^2}{4a^2})-\frac{b^2}{4a}+c$

$y=a(x+\frac{b}{2a})^2-\frac{b^2}{4a}+c$

$p=-\frac{b}{2a}=-\frac{b^2}{4a}+c$

$4ab=2ab^2$

$b=2$

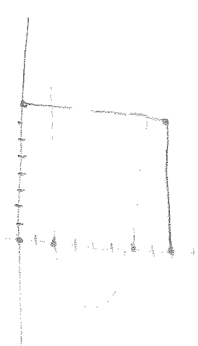
~~14.~~ The parabola $y=x^2-2x+4$ is moved *'p"* units to the right and *"q"* units down. The x-intercepts of the resulting parabola are 3 and 5. What are the values of *"p"* and *"q"*?

SAME AS QUESTION 11

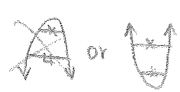
15. Challenge: square OPQR has vertices O(0,0), P(0,8), Q(8,8) and R(8,0). The parabola with equation $y=\overset{a}{\vee}(x-2)(x-6)$ intersects the sides of the square OPQR at points "K", "L", "M", and "N". Determine all the values of "a" for which the area of the trapezoid KLMN is 36.

$y=a(x-2)(x-6) \rightarrow x\ \text{int}=(2,0)\ (6,0)$

$y=ax^2-8ax+12a$



A of S: x=4



or

$\frac{(x+4)8}{2}=36$

$4(x+4)=36$

$x+4=9$

$\boxed{x=5}$

(1.5,8)

(6.5,8)

$y=a(x-2)(x-6)$

$8=a(1.5-2)(1.5-6)$

$8=a(-0.5)(-4.5)$

$8=a(-\frac{1}{2})(-\frac{9}{2})$

$8=\frac{9}{4}a$

$\boxed{a=\frac{32}{9}}$

12. If $y=a(x-2)^2+c$ and $y=(2x-5)(x-b)$ represents the same quadratic function, what is the value of the constant "b"

$y=2x^2-(2b+5)x+5b$

$y=a(x^2-4x+4)+c$

$y=ax^2-4ax+4a+c$

$a=2$

$b=\frac{3}{2}$

13. the parabola $y=ax^2+bx+c$ has vertex $(p,p)$ and y-intercept $(0,-p)$, where $p\neq 0$, what is the value of "b"?   a) $-p$   b) 0   c) 2   D) 4   E) $p$

$-p=c$

$a=\frac{-b}{2p}$

$p=ap^2+bp-p$

$2p=\frac{-bp^2}{2p}+bp$

$4p=-bp+2bp$

$4p=bp$

$4=b$

14. The parabola $y=x^2-2x+4$ is moved *'p"* units to the right and *"q"* units down. The x-intercepts of the resulting parabola are 3 and 5. What are the values of *"p"* and *"q"*?

$y=x^2-2x+4$

$y=(x^2-2x+1)+4-1$

$y=(x-1)^2+3$

vertex $=(1,+3)$

$y=(x-3)(x-5)$

$y=x^2-8x+15$

$y=(x^2-8x+16)+15-16$

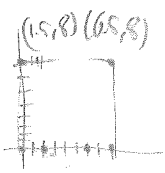
$y=(x-4)^2-1$

vertex $=(4,-1)$

$p=3$

$q=4$

15. Challenge: square OPQR has vertices O(0,0), P(0,8), Q(8,8) and R(8,0). The parabola with equation $y=a(x-2)(x-6)$ intersects the sides of the square OPQR at points "K", "L", "M", and "N". Determine all the values of "a" for which the area of the trapezoid KLMN is 36.




$y=a(x^2-8x+12)$

$y=a(x^2-8x+16)+12a-16a$

$y=a(x-4)^2-4a$

plugin $(1.5,8)$

$4\left(8=a\cdot\frac{25}{4}-4a\right)4$

$32=25a-16a$

$32=9a$

$\frac{32}{9}=a$

$y=\frac{32}{9}(x-4)^2-\frac{128}{9}$

plugin $(6.5,8)$

$8=\frac{32}{9}\cdot\frac{25}{4}-\frac{128}{9}$

$a=\frac{32}{9}$

Name: Ashley Hah Date: Sept 19, 2011

## Section 1.3 Problem Solving Involving Max and Mins

1. Given each of the following equations, find the coordinates of the maximum or minimum point.

| | | |
|---|---|---|
| a) $y=-2x^2+8x$<br>$=-2(x^2-4x)$<br>$=-2(x-2)^2+8$<br>Max $(2,8)$ | b) $y=-4(x-3)(2x-1)$<br>$=-4(2x^2-7x+3)=-8(x^2-\frac{7}{2}x)-12$<br>$=-8(x-\frac{7}{4})^2+\frac{25}{2}$<br>Max $(\frac{7}{4},\frac{25}{2})$ | c) $y=3x^2+9x+12$<br>$=3(x^2+3x)+12$<br>$=3(x+\frac{3}{2})^2+12-\frac{27}{4}$<br>$=3(x+\frac{3}{2})^2+\frac{21}{4}$<br>Min $(-\frac{3}{2},\frac{21}{4})$ |
| d) $y=4x^2+36x+23$<br>$=4(x^2+9x)+23$<br>$=4(x+\frac{9}{2})^2-58$<br>Min $(-\frac{9}{2},-58)$ | e) $y=3(3-2x)^2+7$<br>$=3(4x^2-12x+9)+7$<br>$=12(x^2-3x)+34$<br>$=12(x-\frac{3}{2})^2+7$<br>Min $(\frac{3}{2},7)$ | f) $y=2(x-4)^2+12$<br>$=2(x^2-8x+16)+12$<br>$=2(x-4)^2+44-32$<br>$=2(x-4)^2+12$<br>Min $(4,12)$ |
| g) $y=-2(x-3)^2+8x$<br>$=-2(x^2-6x+9)-2(-4x)$<br>$=-2(x^2-10x+9)$<br>$=-2(x^2-10x)-18$<br>$=-2(x-5)^2+32$<br>Max $(5,32)$ | h) $y=-\frac{3}{2}x^2+15x+2$<br>$=-\frac{3}{2}(x^2-10x)+2$<br>$=-\frac{3}{2}(x-5)^2+\frac{79}{2}$<br>Max $(5,\frac{79}{2})$ | i) $y=-0.75(1-2x)^2+8x$<br>$=-\frac{3}{4}(4x^2-4x+1)-\frac{3}{4}(-\frac{32}{3}x)$<br>$=-\frac{3}{4}(4x^2-\frac{44}{3}x)-\frac{3}{4}$<br>$=-3(x^2-\frac{11}{3}x)-\frac{3}{4}$<br>$=-3(x-\frac{11}{6})^2+\frac{28}{3}$<br>Max $(\frac{11}{6},\frac{28}{3})$ |

2. Two numbers have a difference of 10. Their product is a minimum. Determine the numbers

$x-y=10$

$x=10+y$

$\min = xy = y(10+y) = y^2+10y = (y+5)^2-25$

$xy=(y+5)^2-25 \rightarrow y=-5$

$-5x=-25$ $x=5$

3. The sum of two natural numbers is 12. Their product is a maximum. Determine the numbers

$x+y=12$

$x=12-y$

$\max = xy = y(12-y) = -y^2+12y = -(y^2-12y) = -(y-6)^2+36$

$xy=-(y-6)^2+36 \rightarrow y=6$

$6x=36$ $x=6$

4. The sum of two numbers is 60. Their product is a maximum. Determine the numbers.

$x+y=60$

$x=60-y$

$\max = xy = y(60-y) = -y^2+60y = -(y^2-60y) = -(y-30)^2+900$

$xy=-(y-30)^2+900 \rightarrow y=30$

$30x=900$ $x=30$

5. Two numbers have a difference of 30. The sum of their squares is a minimum. Determine the numbers.

$x-y=30$

$x=30+y$

$\min = x^2+y^2 = y^2+(30+y)^2 = y^2+y^2+60y+900 = 2(y^2+30y)+900 = 2(y+15)^2+450$

$x^2+y^2=2(y+15)^2+450 \rightarrow y=-15$

$x^2+225=450$ $x=15$

6. The sum of two numbers is 32. The sum of their squares is a minimum. Determine the numbers.

$x+y=32$

$x=32-y$

$\min = x^2+y^2 = y^2+(32-y)^2 = y^2+y^2-64y+1024 = 2(y^2-32y)+1024 = 2(y-16)^2+512$

$x^2+y^2=2(y-16)^2+512 \rightarrow y=16$

$x^2+256=512$ $x=16$

7. There is a number such that when you add it to twice its square the sum is minimized. What is this minimum sum?

$\min = x+2x^2 = 2(x^2+\frac{1}{2}x) = 2(x+\frac{1}{4})^2-\frac{1}{8}$

the minimum sum is $-\frac{1}{8}$

8. A Broadway musical sells 400 tickets each day at $30 per ticket. For every increase of $3.00, they lose 20 sales. What should their ticket price be to yield the maximum revenue?

$Q_0 = 400$
$P_0 = 30$
$\Delta Q = -20$
$\Delta P = +3$

$\frac{Q-400}{P-30} = \frac{-20}{3}$

$3Q - 1200 = -20P + 600$

$Q = -\frac{20}{3}P + 600$

revenue $= P(-\frac{20}{3}P + 600) = -\frac{20}{3}(P^2 - 90P) = -\frac{20}{3}(P-45)^2 + 13500$

the ticket price should be $45 per ticket

9. A company that charters a boat for tours around Vancouver Island can sell 200 tickets at $50 each. For every $10 increase in the ticket price, 5 fewer tickets will be sold.

a. Represent the number of tickets sold as a function of the selling price

$\frac{Q-200}{P-50} = \frac{-5}{10}$ $\quad 10Q - 2000 = -5P + 250$

$Q = -\frac{1}{2}P + 225$

b. Represent the revenue as a function of the selling price

$R = P(-\frac{1}{2}P + 225) = -\frac{1}{2}P^2 + 225P = -\frac{1}{2}(P^2 - 450P) = -\frac{1}{2}(P-225)^2 + 25312.5$

c. What selling price will provide the maximum revenue? What is the maximum revenue?

the price of $225 per ticket will provide the maximum revenue

the maximum revenue is $25312.5.

d. What range of price will provide a revenue greater than $20,000?

$-\frac{1}{2}(P-225)^2 + 25312.5 = 20000$

$-\frac{1}{2}(P-225)^2 = -5312.5$

$(P-225)^2 = 10625$

$P - 225 = \pm 25\sqrt{17}$

$P = 225 \pm 25\sqrt{17}$

$121.92 ~ $328.08

10. A company sells its bikes at $300 each and can sell 70 in a season. For every $25 increase in the price, the number sold drops by 10.

a. Represent the sales revenue as a function of the price

$\frac{Q-70}{P-300} = \frac{-10}{25}$ $\quad 25Q - 1750 = -10P + 3000$

$Q = -\frac{2}{5}P + 190$

$R = P(-\frac{2}{5}P + 190) = -\frac{2}{5}P^2 + 190P = -\frac{2}{5}(P^2 - 475P)$

$= -\frac{2}{5}(P - \frac{475}{2})^2 + 22562.5$

b. What price will yield the maximum revenue?

$237.5 will yield the maximum revenue

c. What range of prices will give a sales revenue that exceed $18,000?

$-\frac{2}{5}(P - \frac{475}{2})^2 + 22562.5 = 18000$

$-\frac{2}{5}(P - \frac{475}{2})^2 = -4562.5$

$(P - \frac{475}{2})^2 = 11406.25$

$P - \frac{475}{2} = \pm \frac{25\sqrt{73}}{2}$

$P = \frac{475 \pm 25\sqrt{73}}{2}$

$130.70 ~ $344.30

11. A musical show did a statistical research on the impact of its ticket sold by the ticket price. Through their research, they collected the following data. Using this data, determine what ticket price will yield the maximum revenue.

| Ticket Price | tickets sold |
|---|---|
| $5.00 | 416.67 |
| $6.00 | 400.00 |
| $8.00 | 366.67 |
| $10.00 | 333.33 |
| $12.00 | 300.00 |
| $17.00 | 216.67 |
| $19.00 | 183.33 |

+$1 = -16.67
+$3 = -50
+$5 = -83.34
+$7 = -116.67
+$12 = -200
+$14 = -233.34
x4
x2
there is a pattern.

$Q_0 = 400$
$P_0 = \$6$
$\Delta Q = -50$
$\Delta P = +\$3$

$\frac{Q-400}{P-6} = \frac{-50}{3}$

$3Q - 1200 = -50P + 300$

$Q = -\frac{50}{3}P + 500$

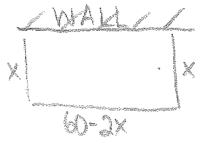
$R = P(-\frac{50}{3}P + 500)$

$= -\frac{50}{3}(P^2 - 30P)$

$= -\frac{50}{3}(P-15)^2 + 3750$

$15 will yield the maximum revenue

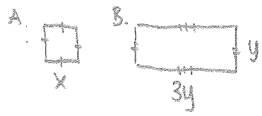
12. A farmer wants to make a rectangular corral by using his barn wall as one of the sides of the corral. If the farmer has only 60m of fence, what length for the rectangular corral would maximize the area?




$max = x(60-2x) = -2x^2+60x = -2(x^2-30x) = -2(x-15)^2+450$

the lengths 15m and 30m would maximize the area //

13. A 50 meter long wire is cut into two separate pieces. One piece is use to make a square and the other into a rectangle, with the length 3 times the width. If the sum of both area is a maximum, find the length of each piece of wire.




$4x+2y+6y=50$

$2x+y+3y=25$

$2x=25-4y$

$x=\frac{25}{2}-2y$

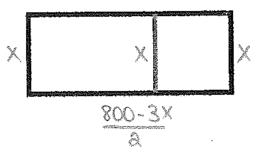
$max = (\frac{25}{2}-2y)^2+3y^2 = 4y^2-50y+\frac{625}{4}+3y^2 = 7(y^2-\frac{50}{7}y)+\frac{625}{4}$

$= 7(y-\frac{50}{14})^2+\frac{625}{4}-\frac{625}{7} = 7(y-\frac{50}{14})^2+\frac{1875}{28} \rightarrow y=\frac{50}{14}$

A. $4x = 4(\frac{25}{2}-\frac{50}{7})$ $= \frac{150}{7}m$ //

B. $8y = 8(\frac{50}{14})$ $= \frac{200}{7}m$ //

14. A rectangular area is enclosed by a fence and separated into 2 rectangular regions as shown. With 800m of fencing, what is the maximum area that could be enclosed. Find the dimensions of the enclosed area.




$max = x(\frac{800-3x}{2}) = \frac{-3x^2+800x}{2} = -\frac{3}{2}(x^2-\frac{800}{3}x) = -\frac{3}{2}(x-\frac{400}{3})^2+\frac{80000}{3}$

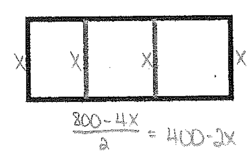
$x=\frac{400}{3}$

the maximum area is $\frac{80000}{3}m^2$

$\frac{800-3x}{2} = \frac{800-400}{2} = 200$

the dimensions are $\frac{400}{3}$m and 200m. //

15. Suppose the rectangular fence is to be separated into 3 rectangular regions as shown. Again, with 800m of fencing, find the maximum area that could be enclosed. Find the dimensions of the enclosed area.




$max = x(400-2x) = -2x^2+400x = -2(x^2-200x) = -2(x-100)^2+20000$

$x=100$

the maximum area is $20000m^2$

$400-2x = 400-200 = 200$

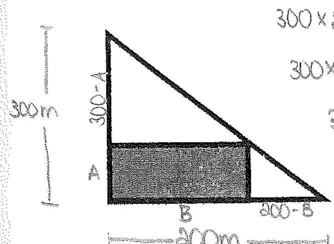
the dimensions are 100m and 200m //

16. Bob is going to start a small rectangular garden using his house as one side and his garage as another side. He has 60m of fencing and wants to enclose a maximum area of 450 square metres. How long will the longest side of his fence be?

* work on seperate sheet

$\therefore 30+15\sqrt{5}$ m //

17. A right triangle, with a base of 200m and height of 300m encloses a rectangle as shown. Find the dimensions of the maximum rectangle? Extension: If the height of the triangle is "A" and the base is "B", find the dimensions of the maximum rectangle in terms of "A" and "B".




$300 \times 200 = 2AB + B(300-A) + A(200-B)$

$300 \times 200 = 2AB + 300B - AB + 200A - AB$

$300 \times 200 = \cancel{2AB} - \cancel{2AB} + 300B + 200A$

$300 \times 2 = 3B + 2A$

$2A = 600-3B$

$A = \frac{600-3B}{2}$

$AB = max = B(\frac{600-3B}{2}) = -\frac{3}{2}B^2+300B = -\frac{3}{2}(B^2-200B)$

$= -\frac{3}{2}(B-100)^2+15000$

$\boxed{B=100}$

$A = \frac{600-300}{2} = \boxed{150}$

the dimensions are 100m and 300m //

* extended part on seperate sheet

18. When $f(x)=ax^2+bx+c$ has a minimum value of "zero", what conditions must be satisfied by $a, b,$ and $c$

$ax^2+bx+c \geq 0$

$ax^2 \geq -bx-c$

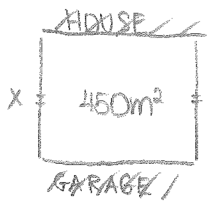
$\boxed{a \geq \frac{-bx-c}{x^2}}$

$bx \geq -ax^2-c$

$\boxed{b \geq \frac{-ax^2-c}{x}}$

$\boxed{c \geq -ax^2-bx}$

Situation A.



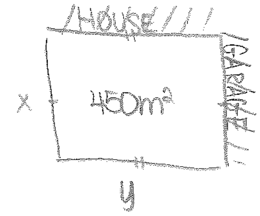

$2x = 60$

$\boxed{x = 30}$

$450 \div 30 = \boxed{15}$ ← not fence!

the longest side of his fence is 30m //

Situation B.




$x + y = 60$

$\boxed{x = 60 - y}$

$\boxed{xy = 450}$

$y(60-y) = 450$

$-y^2 + 60y = 450$

$-(y^2 - 60y) = 450$

$-(y-30)^2 + 900 = 450$

$-(y-30)^2 = -450$

$(y-30)^2 = 450$

$y - 30 = 15\sqrt{2}$

$\boxed{y = 30 + 15\sqrt{2}}$

$x = 60 - (30 + 15\sqrt{2})$

$\boxed{x = 30 - 15\sqrt{2}}$

$y > x$.

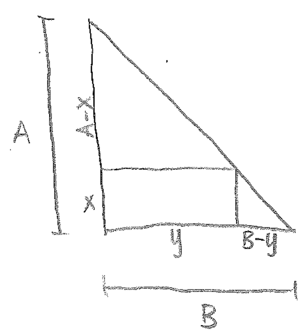
the longest side of the fence is $30 + 15\sqrt{2}$ m //

$30 + 15\sqrt{2} > 30$

∴ situation B has the solution to the question!

Final answer: the longest side of the fence is $30 + 15\sqrt{2}$ m //

#17 cont.




$AB = 2xy + y(A-x) + x(B-y)$

$AB = \cancel{2xy} + Ay - \cancel{xy} + Bx - \cancel{xy}$

$AB = Ay + Bx$

$Ay = AB - Bx$

$\boxed{y = \frac{AB - Bx}{A}}$

$\max = xy = x\left(\frac{AB - Bx}{A}\right) = \frac{ABx - Bx^2}{A} = \frac{1}{A}(-Bx^2 + ABx)$

$= -\frac{B}{A}(x^2 - Ax) = -\frac{B}{A}\left(x - \frac{A}{2}\right)^2 + \frac{AB}{4}$

$\rightarrow \boxed{x = \frac{A}{2}}$

$y = \frac{AB - B\left(\frac{A}{2}\right)}{A} = \left(AB - \frac{BA}{2}\right)\frac{1}{A}$

$= B - \frac{B}{2} = \boxed{\frac{B}{2}}$ //

∴ the dimensions are $\frac{A}{2}$ units and $\frac{B}{2}$ units //